

Hong Kong Mathematics Olympiad (2015/16)
Heat Event (Group)
香港数学竞赛 (2015/16)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 最初甲瓶装有 1 公升酒精，乙瓶是空的。

第 1 次将甲瓶全部的酒精倒入乙瓶中，第 2 次将乙瓶酒精的 $\frac{1}{2}$ 倒回甲瓶，

第 3 次将甲瓶酒精的 $\frac{1}{3}$ 倒入乙瓶，第 4 次将乙瓶酒精的 $\frac{1}{4}$ 倒回甲瓶，……，

第 2016 次后，甲瓶还有多少公升酒精？

At the beginning, there was 1 litre of alcohol in Bottle A and bottle B is an empty bottle.

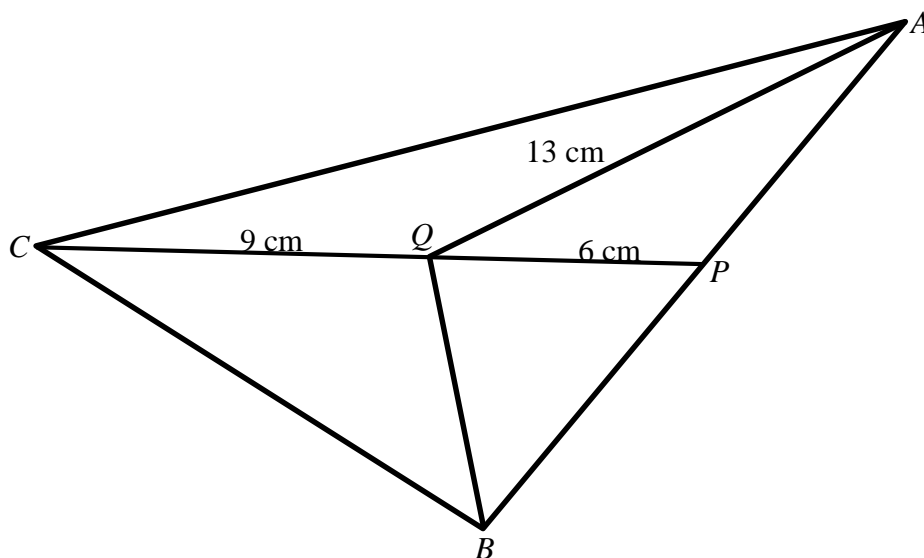
First, pour all alcohol from bottle A to bottle B; second, pour $\frac{1}{2}$ of the alcohol from bottle B back to

bottle A; third, pour $\frac{1}{3}$ of the alcohol from bottle A to bottle B; fourth, pour $\frac{1}{4}$ of the alcohol from bottle

B back to bottle A, After the 2016th pouring, how much alcohol was left in bottle A ?

2. 图一显示 $\triangle ABC$ ， P 为 AB 的中点及 Q 是 CP 上的一点。已知 $BQ \perp CP$ ， $PQ = 6$ cm、 $CQ = 9$ cm 及 $AQ = 13$ cm。求 $\triangle ABC$ 的面积。

Figure 1 shows $\triangle ABC$, P is the mid-point of AB and Q is a point on CP . It is known that $BQ \perp CP$, $PQ = 6$ cm, $CQ = 9$ cm and $AQ = 13$ cm. Find the area of $\triangle ABC$.



图一

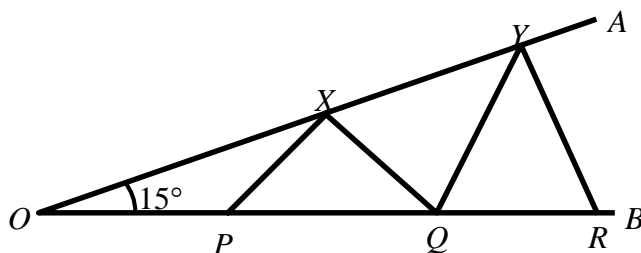
Figure 1

3. 考虑数列 a_1, a_2, a_3, \dots 。定义 $S_n = a_1 + a_2 + \dots + a_n$ 其中 n 为任何整数。若 $S_n = 2 - a_n - \frac{1}{2^{n-1}}$ ，求 a_{2016} 的值。

Consider a sequence of numbers a_1, a_2, a_3, \dots . Define $S_n = a_1 + a_2 + \dots + a_n$ for any positive integer n . Find the value of a_{2016} if $S_n = 2 - a_n - \frac{1}{2^{n-1}}$.

4. 设 x 及 y 为正整数且满足 $\log x + \log y = \log(2x - y) + 1$ ，求 (x, y) 的数量。
If x and y are positive integers that satisfy $\log x + \log y = \log(2x - y) + 1$, find the number of possible pairs of (x, y) .

5. 图二中， $\angle AOB = 15^\circ$ 。X、Y 是 OA 上的点，P、Q、R 是 OB 上的点使得 $OP = 1$ 及 $OR = 3$ 。若 $s = PX + XQ + QY + YR$ ，求 s 的最小值。
In Figure 2, $\angle AOB = 15^\circ$, X, Y are points on OA, P, Q, R are points on OB such that $OP = 1$ and $OR = 3$. If $s = PX + XQ + QY + YR$, find the least value of s .



图二

Figure 2

6. 设 $y = px^2 + qx + r$ 为一二次函数。已知
(1) y 的对称轴为 $x = 2016$
(2) 该函数的图像通过 x 轴于 A、B 两点，其中 $AB = 4$ 单位
(3) 该函数的图像通过直线 $y = -10$ 于 C、D 两点，其中 $CD = 16$ 单位
求 q 的值。

Let $y = px^2 + qx + r$ be a quadratic function. It is known that

- (1) The axis of symmetry of y is $x = 2016$.
(2) The curve cuts the x -axis at two points A and B such that $AB = 4$ units.
(3) The curve cuts the line $y = -10$ at two points C and D such that $CD = 16$ units.

Find the value of q .

7. 设三角形三条中线的长度为 9、12 及 15。求该三角形的面积。

The lengths of the three medians of a triangle are 9, 12 and 15. Find the area of the triangle.

8. 若某正整数的二进制表示有以下特质：

(1) 有 11 个位，

(2) 有六个位是 1，有五个位是零，

则称该数为「好数」。

(例如：2016 是一个「好数」，因为 $2016 = 11111100000_2$ 。)

求所有「好数」的和。

If the binary representation of a positive integer has the following properties:

(1) the number of digits = 11,

(2) the number of 1's = 6 and the number of 0's = 5,

then the number is said to be a “good number”.

(For example, 2016 is a “good number” as $2016 = 11111100000_2$.)

Find the sum of all “good numbers”.

9. 设整数 a 、 b 及 c 为三角形的边长。已知 $f(x) = x(x-a)(x-b)(x-c)$ ，且 x 为一个大于 a 、 b 及 c 的整数。若 $x = (x-a) + (x-b) + (x-c)$ 及 $f(x) = 900$ ，求该三角形三条垂高的总和。

Let the three sides of a triangle are of lengths a , b and c where all of them are integers. Given that $f(x) = x(x-a)(x-b)(x-c)$ where x is an integer of size greater than a , b and c . If $x = (x-a) + (x-b) + (x-c)$ and $f(x) = 900$, find the sum of the lengths of the three altitudes of this triangle.

10. 求 $\frac{1^4 + 2015^4 + 2016^4}{1^2 + 2015^2 + 2016^2}$ 的值。

Find the value of $\frac{1^4 + 2015^4 + 2016^4}{1^2 + 2015^2 + 2016^2}$.

完
END